Existence and non-existence of maximizers for the Moser-Trudinger inequalities of the inhomogeneous type

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Abstract
In this talk, we consider the existence and non-existence of maximizers for the Moser-Trudinger type inequalities stating

$$D_{N,\alpha,\gamma} := \sup_{u \in H^{1,1}(\mathbb{R}^N), \|u\|_{H^{1,1}(\mathbb{R}^N)} = 1} \int_{\mathbb{R}^N} \Phi_N \left( \alpha |u|^{N'} \right) \, dx < +\infty$$

for $\alpha \leq \alpha_N := N^2 \frac{1}{N-1}$, where $N \geq 2$, $N' = \frac{N}{N-1}$, $\Phi_N(t) = \sum_{j=N-1}^{\infty} \frac{t^j}{j!}$ and $\|u\|_{H^{1,1}(\mathbb{R}^N)}^\gamma = \|u\|^\gamma + \|\nabla u\|^\gamma$. We clarify the effect of $\gamma$ on the existence of maximizers for $D_{N,\alpha,\gamma}$, and show that $D_{N,\alpha,\gamma}$ admits a maximizer for all $\alpha < \alpha_N$ when $\gamma > N'$, while $D_{N,\alpha,\gamma}$ is never attained for any sufficiently small $\alpha$ when $\gamma \leq N'$. This result is joint work with Professor Michinori Ishiwata in Osaka University and Professor Norihisa Ikoma in Kanazawa University.

References


