Existence and non-existence of maximizers for the Moser-Trudinger inequalities of the inhomogeneous type

Hidemitsu Wadade
Institute of Science and Engineering, Kanazawa University

Abstract

In this talk, we consider the existence and non-existence of maximizers for the Moser-Trudinger type inequalities stating

$$D_{N,\alpha,\gamma} := \sup_{u \in H^{1,N}(\mathbb{R}^N),\, \|u\|_{H^{1,N}_\gamma}=1} \int_{\mathbb{R}^N} \Phi_N\left(\alpha |u|^{N'}\right) dx < +\infty$$

for $\alpha \le \alpha_N := N\omega_{N-1}^{\frac{1}{N-1}}$, where $N \ge 2$, $N' = \frac{N}{N-1}$, $\Phi_N(t) = \sum_{j=N-1}^{\infty} \frac{t^j}{j!}$ and $\|u\|^\gamma_{H^{1,N}_\gamma} = \|u\|^\gamma_N + \|\nabla u\|^\gamma_N$. We clarify the effect of γ on the existence of maximizers for $D_{N,\alpha,\gamma}$, and show that $D_{N,\alpha,\gamma}$ admits a maximizer for all $\alpha < \alpha_N$ when $\gamma > N'$, while $D_{N,\alpha,\gamma}$ is never attained for any sufficiently small α when $\gamma \le N'$. This result is joint work with Professor Michinori Ishiwata in Osaka University and Professor Norihisa Ikoma in Kanazawa University.

References

[1] L. Carleson and S.-Y. A. Chang, *On the existence of an extremal function for an inequality of J. Moser*, Bull. Sci. Math. (2) **110** (1986), 113–127.

[2] M. Flucher, *Extremal functions for the Trudinger-Moser inequality in 2 dimensions*, Comm. Math. Helv. **67** (1992), 471–479.

[3] M. Ishiwata, *Existence and nonexistence of maximizers for variational problems associated with Trudinger-Moser type inequalities in* $\mathbb{R}^N$, Math. Ann. **351** (2011), 781–804.

[4] Y. Li and B. Ruf, *A sharp Trudinger-Moser type inequality for unbounded domains in* $\mathbb{R}^n$, Indiana Univ. Math. J. **57** (2008), 451–480.

[5] K. C. Lin, *Extremal functions for Moser's inequality*, Trans. Amer. Math. Soc. **348** (1996), 2663–2671.

[6] J. Moser, *A sharp form of an inequality by N. Trudinger*, Indiana Univ. Math. J. **20** (1970), 1077–1092.

[7] B. Ruf, *A sharp Trudinger-Moser type inequality for unbounded domains in* $\mathbb{R}^2$, J. Funct. Anal. **219** (2005), 340–367.