半導体の熱伝導流体力学モデルの定常解の漸近安定性

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We discuss the existence and the asymptotic stability of a stationary solution to the initial boundary value problem for a one-dimensional heat–conductive hydrodynamic model for semiconductors. It is given by the system of equations

$$\rho_t + j_x = 0, \tag{1a}$$

$$j_t + \left(\frac{j^2}{\rho} + p(\rho,\theta)\right)_x = \rho\phi_x - \frac{j}{\tau_m}, \tag{1b}$$

$$\rho\theta_t + j\theta_x + \frac{2}{3}\left(\frac{j}{\rho}\right)_x \rho\theta - \frac{2}{3}(\kappa\theta_x)_x = \nu\frac{j^2}{\rho} - \frac{\rho}{\tau_e}(\theta - \bar{\theta}), \tag{1c}$$

$$\phi_{xx} = \rho - D, \tag{1d}$$

for $x \in \Omega := (0,1)$. Here, the unknown functions ρ, j, θ and ϕ stand for electron density, electric current, absolute temperature and electrostatic potential, respectively. The positive constants $\bar{\theta}$, κ, τ_m and τ_e mean ambient device temperature, thermal conductivity, momentum relaxation time and energy relaxation time, respectively. Moreover, the constant ν is equal to $(2\tau_e - \tau_m)/\tau_m\tau_e$. The pressure p is assumed that

$$p = p(\rho,\theta) = \rho\theta. \tag{2}$$

The doping profile D is a bounded continuous function of the spatial variable x and satisfy

$$\inf_{x\in(0,1)} D(x) > 0. \tag{3}$$

We prescribe the initial and the boundary data as

$$(\rho, j, \theta)(0,x) = (\rho_0, j_0, \theta_0)(x), \tag{4}$$

$$\rho(t,0) = \rho_l > 0, \quad \rho(t,1) = \rho_r > 0, \tag{5}$$

$$\theta(t,0) = \theta_l > 0, \quad \theta(t,1) = \theta_r > 0, \tag{6}$$

$$\phi(t,0) = 0, \quad \phi(t,1) = \phi_r > 0, \tag{7}$$

where ρ_l, ρ_r, ϕ_r, θ_l and θ_r are constants. It is also assumed that the initial data (4) is compatible with the boundary data (5)–(7). Namely, $\rho_0(0) = \rho_l$, $\rho_0(1) = \rho_r$, $j_{0x}(0) = j_{0x}(1) = 0$, $\theta_0(0) = \theta_l$ and $\theta_0(1) = \theta_r$. We study this problem in the region where the subsonic condition (8a) and positivity of the density and the temperature (8b) hold, that is,

$$\inf_{x\in\Omega}\left(\theta - \frac{j^2}{\rho^2}\right)(t,x) > 0, \tag{8a}$$

$$\inf_{x\in\Omega}\rho(t,x) > 0, \quad \inf_{x\in\Omega}\theta(t,x) > 0. \tag{8b}$$

Hence, we need to assume the initial data satisfies this condition:

$$\inf_{x\in\Omega}\left(\theta_0-\frac{j_0^2}{\rho_0^2}\right)(x)>0,\quad \inf_{x\in\Omega}\rho_0(x)>0,\quad \inf_{x\in\Omega}\theta_0(x)>0. \tag{9}$$

The subsonic condition implies one characteristic speed of the hyperbolic equations (1a) and (1b) is negative and another is positive. Namely,

$$\lambda_1:=u-\sqrt{\theta}<0,\quad \lambda_2:=u+\sqrt{\theta}>0, \tag{10}$$

where $u:=j/\rho$ stands for velocity of electron flow. Therefore we see that three boundary conditions (5)–(7) are necessary and sufficient for the well–posedness of this initial boundary value problem.

The purpose of the present talk is to show the asymptotic stability of a stationary solution, which is a solution to (1) independent of time t, satisfying the same boundary conditions (5)–(7). Hence, the stationary solution $(\tilde{\rho},\tilde{j},\tilde{\phi})$ satisfies the equations

$$\tilde{j}_x=0, \tag{11a}$$

$$\left(\tilde{\theta}-\frac{\tilde{j}^2}{\tilde{\rho}^2}\right)\tilde{\rho}_x+\tilde{\rho}\tilde{\theta}_x=\tilde{\rho}\tilde{\phi}_x-\frac{\tilde{j}}{\tau_1}, \tag{11b}$$

$$\tilde{j}\tilde{\theta}_x+\frac{2}{3}\left(\frac{\tilde{j}}{\tilde{\rho}}\right)_x\tilde{\rho}\tilde{\theta}-\frac{2}{3}\kappa\tilde{\theta}_{xx}=\nu\frac{\tilde{j}^2}{\tilde{\rho}}-\frac{\tilde{\rho}}{\tau_2}(\tilde{\theta}-\bar{\theta}), \tag{11c}$$

$$\tilde{\phi}_{xx}=\tilde{\rho}-D. \tag{11d}$$

with the conditions (5)–(7).

The hydrodynamic model for semiconductors draws a lot of attentions from mathematicians for these decades. Especially, isentropic model is intensively studied (see [2, 4, 6]). Among them, the asymptotic stability of the stationary solution was proved in [3, 7]. On the other hand, the heat-conductive model is not studied so much, although it is important to handle a hot carrier problem, which is annoying issue to make the semiconductor devices unstable. The present research studies the heat-conductive model and shows the global solvability and the asymptotic stability of the stationary solution, which were open problems under significant settings from the physical point of view.

The existence of the stationary solution is summarized in the next lemma.

Lemma 1. *Let the doping profile and the boundary data satisfy conditions* (3) *and* (5)*–*(7)*. For arbitrary* ρ_l *and* $\bar{\theta}$*, there exits a positive constant* δ_1 *such that if* $\delta:=|\rho_r-\rho_l|+|\theta_r-\bar{\theta}|+|\theta_l-\bar{\theta}|+|\phi_r|\le\delta_1$*, then the stationary problem* (11) *and* (5)*–*(7) *has a unique solution* $(\tilde{\rho},\tilde{j},\tilde{\theta},\tilde{\phi})$ *satisfying the conditions* (8) *in the space* $\mathcal{B}^2(\overline{\Omega})$.

The proof of the existence of the stationary solution $(\tilde{\rho}, \tilde{j}, \tilde{\phi})$ is given by the Schauder and the Leray–Schauder fixed point theorems. The uniqueness follows from the elementary energy method.

We consider the nonstationary problem in the function spaces:

$$\mathfrak{X}_i^j([0,T]) := \bigcap_{k=0}^{i} C^k([0,T]; H^{j+i-k}(\Omega)), \quad \mathfrak{X}_i([0,T]) := \mathfrak{X}_i^0([0,T]) \quad \text{for} \quad i,\ j = 0,1,2,$$
$$\mathfrak{Y}([0,T]) := C([0,T]; H^2(\Omega)) \cap C^1([0,T]; L^2(\Omega)).$$

The satability theorem of the stationary solution is stated as follows.

Theorem 2. *Let* $(\tilde{\rho}, \tilde{j}, \tilde{\phi})$ *be the stationary solution of* (11) *and* (5)–(7). *Suppose that the initial data* $(\rho_0, j_0, \theta_0) \in H^2(\Omega)$ *and the boundary data* ρ_l, ρ_r, θ_l, θ_r *and* ϕ_r *satisfy* (5)–(7), (9) *and the compatibility condition. Then there exits a positive constant* δ_2 *such that if* $\delta + \|(\rho_0 - \tilde{\rho}, j_0 - \tilde{j}, \theta_0 - \tilde{\theta})\|_2 \leq \delta_2$, *the initial boundary value problem* (1), (4) *and* (5)–(7) *has a unique solution* $(\tilde{\rho}, j, \theta, \phi)$ *satisfying* $\rho \in \mathfrak{X}_2([0,\infty))$, $j \in \mathfrak{X}_1^1([0,\infty))$, $\theta \in \mathfrak{Y}([0,\infty)) \cap H^1_{loc}(0,\infty; H^1)$ *and* $\phi \in C^2([0,\infty); H^2)$. *Moreover, the solution* (ρ, j, θ, ϕ) *verifies the regularities* $j_{tt} \in L^2(0,\infty; L^2)$ *and* $\phi - \tilde{\phi} \in \mathfrak{X}_2^2([0,\infty))$ *and the decay estimate*

$$\|(\rho - \tilde{\rho}, j - \tilde{j}, \theta - \tilde{\theta})(t)\|_2 + \|(\phi - \tilde{\phi})(t)\|_4 \leq C\|(\rho_0 - \tilde{\rho}, j_0 - \tilde{j}, \theta_0 - \tilde{\theta})\|_2 e^{-\alpha t}, \quad (12)$$

where C *and* α *are positive constants independent of a time variable* t.

In the proof of Theorem 2, we first obtain the elliptic estimate, and then we construct the unique existence of the time local solution by using an iteration method. Next, an energy form is introduced in order to obtain the basic estimate. Moreover, we apply the energy method to the system of the equations for the perturbation from the stationary solution to get the higher order estimates. Thus, an a-priori estimate is obtained. Then the existence of the time global solution follows from the combination of the existence of the time local solution and the a-priori estimate. Finally, the decay estimate (12) is shown by the a-priori estimate thus obtained.

For the detailed proof of these results, please see the paper [9].

Notation. For a nonnegative integer $l \geq 0$, $H^l(\Omega)$ denotes the l-th order Sobolev space in the L^2 sense, equipped with the norm $\|\cdot\|_l$. We note $H^0 = L^2$ and $\|\cdot\| := \|\cdot\|_0$. $C^k([0,T]; H^l(\Omega))$ denotes the space of the k-times continuously differentiable functions on the interval $[0,T]$ with values in $H^l(\Omega)$. $H^k(0,T; H^l(\Omega))$ is the space of H^k–functions on $(0,T)$ with values in $H^l(\Omega)$. For a nonnegative integer $k \geq 0$, $\mathcal{B}^k(\overline{\Omega})$ denotes the space of the functions whose derivatives up to k-th order are continuous and bounded over $\overline{\Omega}$.

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