Semi-classical analysis and Wigner measure for non-relativistic QED

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This is a joint work with Zied Ammari and Marco Falconi.

We consider a *semi-classical limit* of the Hamiltonian of the non-relativistic QED $H_\hbar$ which is defined on $L^2(\mathbb{R}^d_x) \otimes \mathcal{F}_{\rm rad} \cong L^2(\mathbb{R}^d_x; \mathcal{F}_{\rm rad})$. Let $A_\varphi(x)$ be a quantized radiation field with ultraviolet cutoff φ and $H_{\rm field} = \sum_{j=1}^{d-1} \int |k| a^*(k,j) a(k,j) dk$ the free field Hamiltonian. $H_\hbar$ is given by

$$H_\hbar = \frac{1}{2}(-i\hbar\nabla_x \otimes 1 + \sqrt{\hbar} A_\varphi(x))^2 + V \otimes 1 + \hbar 1 \otimes H_{\rm field}$$

We derive *Wigner measures* μ_t, $t \geq 0$, on

$$X = \mathbb{R}^d \oplus \mathbb{R}^d \oplus L^2(\mathbb{R}^d; \mathbb{C}^{d-1})$$

associated to $H_\hbar$ as $\hbar$ (we take a subsequence) goes to zero. The flow $\Phi_t : X \to X$ on the phase space gives

$$\Phi_t \# \mu_0 = \mu_t,$$

and the classical Maxwell-Newton equation with cutoff φ is derived.